Two Methods for Construction Bidding: Arithmetic Summation vs. Statistical Modeling by Spec Section

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October 22, 2025

Abstract

This paper presents two explicit, step-by-step workflows for estimating building-envelope scope (storefront, curtain wall, glazing, hardware). Method A is the industry-standard *arithmetic estimate*: collect quotes and add them. Method B is a *statistical estimate*: encode early information (spec sections, square footage, and initial vendor quotes) and predict total and phase-level costs using historical correlations. We define input schemas, model forms, validation procedures, and reporting rules for significant figures.

1 Problem Statement

Given early-stage artifacts (specifications, drawings) and initial quotes for glass/metal/hardware, estimate: (i) total project cost, and (ii) a breakdown by job cost codes (JCCs). Let $s \in \mathcal{S}$ index spec sections (e.g., 08 41 13 storefront; 08 44 13 curtain wall; 08 80 00 glazing).

2 Method A: Arithmetic Summation (Current Practice)

A.1 Inputs

1. Architectural spec sections and drawings.
2. Quantity takeoff per spec section: square footage SF_s, counts, lengths.
3. Vendor quotes: two per commodity per section (glass, metal, hardware). Denote quotes $q_{s,1}^{(g)}, q_{s,2}^{(g)}, q_{s,1}^{(m)}, q_{s,2}^{(m)}, q_{s,1}^{(h)}, q_{s,2}^{(h)}$.

A.2 Procedure

1. Choose one quote per commodity per section (often the minimum): $\tilde{q}_s^{(c)} = \min\{q_{s,1}^{(c)}, q_{s,2}^{(c)}\}$ for $c \in \{g, m, h\}$.

2. Add allowances (sealants, sundries), add labor guess, add markup.
3. Sum across sections to produce bid: $\hat{C}_{\text{arith}} = \sum_{s \in \mathcal{S}} \left(\tilde{q}_s^{(g)} + \tilde{q}_s^{(m)} + \tilde{q}_s^{(h)} \right) + \text{allowances} + \text{labor} + \text{markup}$.

A.3 Properties

Digits imply false precision. No uncertainty or correlation is modeled; phase-level forecasts are ad hoc.

3 Method B: Statistical Modeling (Proposed)

B.1 Input Schema (per spec section s)

- Quantities: SF_s, counts, mullion length.
- Quotes (two each): $q_{s,1}^{(g)}, q_{s,2}^{(g)}$; $q_{s,1}^{(m)}, q_{s,2}^{(m)}$; $q_{s,1}^{(h)}, q_{s,2}^{(h)}$.
- Encodings: one-hot vector $\phi(s)$ for spec section class; optional project covariates (region, union/non-union, schedule).

Define engineered features per section:

$$Q_{s,\text{mean}}^{(c)} = \tfrac{1}{2}\big(q_{s,1}^{(c)} + q_{s,2}^{(c)}\big), \qquad Q_{s,\text{min}}^{(c)} = \min\{q_{s,1}^{(c)}, q_{s,2}^{(c)}\}, \qquad Q_{s,\text{spread}}^{(c)} = \big|q_{s,1}^{(c)} - q_{s,2}^{(c)}\big|. \tag{1}$$

B.2 Targets

From historical jobs $j = 1, \ldots, n$, extract actuals:

- Total cost C_j.
- Phase breakdown by job cost code (vector): $\mathbf{y}_j \in \mathbb{R}^K$ (e.g., materials, labor, sealant, equipment, overhead; K codes).

Aggregate by spec section: for job j with sections $s \in \mathcal{S}_j$, define section feature vectors $\mathbf{x}_{j,s}$ as concatenation of $\phi(s)$, SF_s, and engineered quote statistics. Summed drivers for job j:

$$\mathbf{X}_j = \sum_{s \in \mathcal{S}_j} \mathbf{x}_{j,s}. \tag{2}$$

B.3 Models

(i) Total-cost model Regularized linear model (ridge/LASSO):

$$C_j = \beta_0 + \boldsymbol{\beta}^\top \mathbf{X}_j + \varepsilon_j, \qquad \varepsilon_j \sim \mathcal{N}(0, \sigma^2). \tag{3}$$

(ii) Phase (multi-output) model Predict JCC vector with a shared design matrix:

$$\mathbf{y}_j = \mathbf{B}\,\mathbf{X}_j + \boldsymbol{\epsilon}_j, \qquad \boldsymbol{\epsilon}_j \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}). \tag{4}$$

Here $\mathbf{B} \in \mathbb{R}^{K\times p}$ maps drivers to codes, and $\boldsymbol{\Sigma}$ captures inter-code covariance (e.g., glass up $\rightarrow$ sealant up).

(iii) Hierarchical refinement (optional) Treat spec sections as random effects: add $u_{\text{section}(s)}$ to capture persistent deltas across sections/vendors.

B.4 Estimation and Validation

1. Fit on historical jobs with k-fold cross-validation; tune regularization to minimize RMSE/MAPE.
2. Calibrate uncertainty by retaining residual variance and (for multi-output) the empirical $\boldsymbol{\Sigma}$.
3. Backtest on hold-out projects: compare statistical predictions to arithmetic sums and to realized costs.

B.5 Prediction Workflow (New Bid)

1. Parse spec sections and takeoff to build $\{\text{SF}_s\}$ and quotes $\{q_{s,\cdot}^{(\cdot)}\}$.
2. Compute features $\mathbf{x}_s$ per section and sum to $\mathbf{X} = \sum_s \mathbf{x}_s$.
3. Predict total: $\hat{C} = \hat{\beta}_0 + \hat{\boldsymbol{\beta}}^\top \mathbf{X}$ with CI: $\hat{C} \pm 1.96\,\hat{\sigma}\,\sqrt{1 + \mathbf{X}^\top(\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}}$.
4. Predict phase vector: $\hat{\mathbf{y}} = \hat{\mathbf{B}}\,\mathbf{X}$ with covariance $\hat{\boldsymbol{\Sigma}}$.
5. Report significant figures based on signal-to-noise: round $\hat{C}$ and each code in $\hat{\mathbf{y}}$ to match the 95% CI width (typically 2 sig figs).

4 Comparison

- **Arithmetic**: fast, simple, implies false precision; cannot learn from history; phase breakdown is manual.
- **Statistical**: learns correlations between {spec section, SF, quote statistics} and actual costs; yields calibrated totals and JCC breakdowns with uncertainty.

5 Implementation Checklist

1. Data spec: per job, store $(s, \mathrm{SF}_s, q^{(g)}_{s,1:2}, q^{(m)}_{s,1:2}, q^{(h)}_{s,1:2})$, final C, and $\mathbf{y}$ by code.

2. Feature pipeline: compute means/mins/spreads; one-hot spec sections; optional region/schedule flags.

3. Modeling: fit total-cost ridge; fit multi-output least squares for codes; retain residuals/covariances.

4. Reporting: always include % error bands and use significant figures consistent with CI widths.

6 Worked Example (Schema)

Suppose storefront: SF = 1000, quotes average to $Q^{(g)}_{\text{mean}} = \$\,60/\text{ft}^2$, $Q^{(m)}_{\text{mean}} = \$\,40/\text{ft}^2$, $Q^{(h)}_{\text{mean}} = \$\,20/\text{ft}^2$. The arithmetic bid totals \$ 120,000. The statistical model, trained on prior jobs, yields $\hat{C} = \$\,138{,}000 \pm \$\,18{,}000$ with code breakdown (materials, labor, sealant, equipment, OH): $(78, 45, 5, 4, 6)$ k. Report as 1.38×10^5 (two significant figures), not 138,013.

7 Conclusion

Listing and summing quotes is not a measurement model. Statistical estimation by spec section—using engineered quote features and square footage—produces predictive totals, phase-level allocations, and honest precision. Both can be shown side-by-side, but only the statistical method generalizes and improves with data.